AIAA JOURNAL
Vol. 38, No. 6, June 2000

Assessment and Modification of Two-Equation Turbulence Models

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Four turbulence models, Baldwin–Lomax (Baldwin, B., and Lomax, H., "Thin-Layer Approximation and Algebraic Model for Separated Turbulent Flows," AIAA Paper 78-257, 1978), Dash et al. k–ϵ, (Dash, S. M., Beddini, R. A., Wolf, D. E., and Sinha, N., "Viscous/Inviscid Analysis of Curved Sub- or Supersonic Wall Jets," AIAA Paper 83-1679, 1983), Wilcox k–ω (Wilcox, D. C., "Reassessment of Scale-Determining Equation for Advanced Turbulence Models," *AIAA Journal*, Vol. 26, No. 11, 1988, pp. 1299–1310), and Wilcox multiscale (Wilcox, D. C., "Multiscale Model for Turbulent Flows," *AIAA Journal*, Vol. 26, No. 11, 1998, pp. 1311–1320), have been assessed for predicting development of plane and curved turbulent wall-jet flows. The k–ω and multiscale models were also modified to improve their performance for wall jets. In their original form, all of the models exhibited significant deficiencies, although all performed reasonably well for wall jets in an external stream of relatively high velocity. The Dash et al. k–ϵ model performed reasonably well in an overall sense, particularly for curved wall jets in still air. The deficiencies of the other three models were particularly severe for these flows. After modification, the k–ω model gave reasonably good predictions, except for C_f. The multiscale model, even after modification, gave poor predictions of velocity profiles for both plane and curved still-air wall jets and also poor predictions of C_f for curved still-air wall jets. Unfortunately, the modifications were not universal; that is, optimum values of the model "constants" varied substantially from one flow to another. Either the Baldwin–Lomax model, with an outer-layer curvature correction, or a k–ϵ model similar to that of Dash et al. appears to be most suitable at this time for engineering calculations involving wall-jet flows.

Nomenclature

b	= slot height
C_f	= skin-friction coefficient, $\tau_w/0.5\rho U_m^2$
e	= kinetic energy of lower partition eddies
h	= $1 + y/R$
k	= turbulence kinetic energy
p	= static pressure
R	= radius of curvature of surface (positive for convex)
U_e, U_m	= edge velocity and maximum velocity; see Fig. 1
u, v	= time average x and y velocity components
x, y	= surface-streamwise and normal coordinates; see Fig. 1
$y_{1/2}$, y_m	= wall jet thickness parameters; see Fig. 1
y^+	= yu_τ/ν where u_τ is $(\tau_w/\rho)^{1/2}$
α	= empirical curvature factor; Eqs. (1) and (2)
ϵ	= dissipation rate of turbulence kinetic energy
ν, ν_t	= kinematic viscosity and eddy viscosity
ρ	= fluid density
τ_w	= wall shear stress
ω	= $\epsilon/\beta^* k$, where $\beta^* = 0.09$ is a model constant
$'$	= fluctuation velocity components
$\bar{\ }$	= time averaging

Introduction

TURBULENCE models are a crucial element in Navier–Stokes computations of viscous flows. Experience has shown that turbulence models are not universal and must be adjusted or tuned to suit different types of flow. This paper reports on efforts to assess and adapt existing turbulence models to computation of two-dimensional plane and curved wall-jet flows (see Fig. 1). Launder and Rodi[1] review characteristics and behavior of turbulent wall jets.

Wall-jet flows are of considerable engineering importance. For example, they appear where tangential blowing from a slot is used to provide film cooling or to delay or to prevent flow separation in diffusers or on trailing-edge flaps. Another interesting application is circulation control,[2,3] where the airfoil section has a rounded, rather than a sharp, trailing edge, and tangential blowing is used to move the rear stagnation point around this trailing edge. Large values of lift coefficient are attainable, and lift can be varied independently of incidence. In circulation control, and in most other aerodynamic applications, the wall jets are highly curved. Accurate predictions of the wall-jet region of the flowfield are crucial to accurate prediction of circulation-controlled airfoil performance.

Despite their importance, the literature contains relatively few attempts to apply and assess modern turbulence models for computation of wall-jet flows. For example, only two groups reported computational results for wall jets at the 1980–1981 Stanford conference.[4] Early prediction methods were based on integral methods.[5–7] Finite difference methods were first applied to wall jets in the early 1970s, for example, Kacker and Whitelaw,[8] Pai and Whitelaw,[9] Hanjalic and Launder,[10] and Dvorak.[11] Kacker and Whitelaw[8] used the turbulence kinetic energy equation together with a van-Driest-like[12] relation for the variation of the turbulence length scale across the shear layer, whereas Pai and Whitelaw[9] used a mixing length turbulence model, with ad hoc expressions for the mixing length. Hanjalic and Launder[10] applied a Reynolds stress transport (RST) model to numerous thin shear flows, including a plane still-air wall jet. Dvorak[11] used various ad hoc correlations to determine the eddy viscosity in different zones of the wall jet. These early investigations showed reasonably good agreement with experiment, but the range of flows considered was somewhat limited; in particular only Dvorak considered strongly curved wall jets. As reported by Launder and Rodi[1] in their 1983 review, by the early 1980s it was apparent that mixing-length and two-equation turbulence models, which rely on the eddy viscosity hypothesis, require large corrections to deal with curved wall jets whereas RST and algebraic stress models (ASM) can capture curvature effects naturally, that is, without introduction of curvature-correction factors. However Launder and Rodi also point out that predictions using ASM that rely on the k and

Presented as Paper 99-0790 at the AIAA 37th Aerospace Sciences Meeting, Reno, NV, 11–14 January 1999; received 9 March 1999; revision received 10 May 1999; accepted for publication 20 October 1999.

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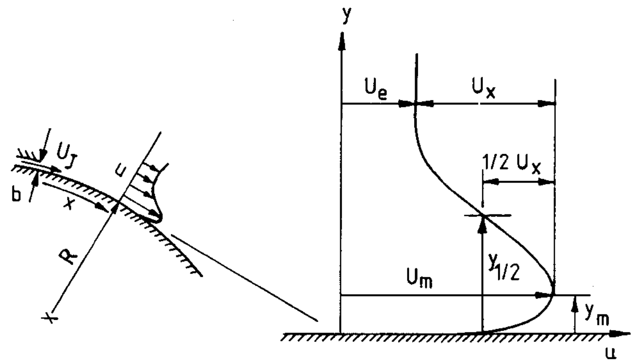

Fig. 1 Definition of a wall-jet flow.

ϵ transport equations are strongly dependent on the correlations used to represent wall damping and the somewhat controversial (see Ref. 13, 2nd ed., p. 290) pressure echo effects. A two-equation turbulence model specifically tuned for curved wall jets was presented by Dash et al.[14] in 1983; it is a hybrid two-layer model that combines a van Driest[12] inner-layer formulation with a curvature-corrected high-Reynolds-number k–ϵ model[15] for the outer layer of the wall-jet flow. Their model yielded good agreement with flowfield data for wall jets. Since 1990 a number of authors have presented Navier–Stokes computations for circulation controlled airfoils, using curvature corrected Baldwin–Lomax[16] and/or k–ϵ turbulence models. Of these authors, only Holz et al.,[17] who used a modified Baldwin–Lomax model, presented comparisons of predictions with data for the wall-jet region of the flowfield.

The present work assessed four turbulence models, the Baldwin–Lomax,[16] Dash et al. k–ϵ (Ref. 14), Wilcox k–ω (Ref. 18), and Wilcox multiscale[19] models. The work was limited to the two-equation and simpler class of turbulence models because they are widely used and computations with them are sufficiently fast for engineering purposes. The main effort was devoted to the k–ω and multiscale (MS) models because they have seen little application to wall-jet flows and because the Wilcox k–ω (Ref. 18) model has demonstrated superior performance for conventional boundary layers in adverse pressure gradients, whereas the MS model does not rely on the eddy-viscosity hypothesis that gives zero shear stress at the velocity peak, that is at $y = y_m$ in Fig. 1. It is well known that the zero-shear-stress point in wall jets occurs substantially closer to the wall than the velocity peak. The Baldwin–Lomax[16] and k–ϵ models were included in the assessment because of their popularity; the Dash et al.[14] form of the latter was used because modifications for curvature are already incorporated and because its inner-layer formulation avoids the serious deficiencies of the standard k–ϵ model in adverse pressure gradients (Ref. 13, Chap. 4).

As will be seen, in their original form none of the turbulence models was entirely satisfactory for wall jets. As mentioned, the k–ω and MS models were the main focus of the work, and modifications to these two methods were developed, using an approach outlined by Wilcox,[13] in an attempt to improve their performance for wall jets.

Turbulence Models

As already mentioned, four turbulence models were assessed and two of them were modified to improve their performance for wall-jet flows. Brief descriptions of the unmodified versions of the models, with emphasis on their minor differences from the basic versions in the references, are presented here.

The Baldwin–Lomax[16] model is an algebraic eddy-viscosity model that uses a mixing length formulation, with van Driest[12] damping very near the wall, to determine the eddy viscosity in the inner zone of the shear layer. In the outer zone, the eddy viscosity is constant with y except near the outer edge, where it is modulated by an intermittency function. The outer zone begins where the eddy viscosity given by the inner-zone expression exceeds that for the outer zone. The model used in the present work was identical to that in Ref. 16, with one exception. Following Shrewsbury,[20] the mixing length in the inner zone of the model was multiplied by the curvature correction factor

$$F_c = 1 - \alpha\left(\frac{u}{R + y}\right)\Big/\left(\frac{\partial u}{\partial y}\right) \tag{1}$$

where R is the radius of curvature of the surface, positive for convex surfaces, and α is an empirical factor, set to 25. In keeping with Shrewsbury's work, no curvature correction was applied in the outer zone.

The Dash et al.[14] turbulence model uses a mixing length model near the wall and a standard high-Reynolds-number k–ϵ model,[15] with a curvature correction, away from the wall. The basic mixing length model is identical to that in the Baldwin–Lomax[16] model except that $\partial u/\partial y$ is used instead of the approximate vorticity, $[\partial u/\partial y + u/(R + y)]$. The mixing length is multiplied by the curvature correction factor

$$F_c = \left(1 - \alpha\left(\frac{u}{R}\right)\Big/\left(\frac{\partial u}{\partial y}\right)\right)\Big/\left(1 - \left(\frac{u}{R}\right)\Big/\left(\frac{\partial u}{\partial y}\right)\right) \tag{2}$$

Dash et al.[14] suggested α values in the range 5–10; we used 5. In the coordinates of Fig. 1, the k and ϵ equations are as follows:

$$u\frac{\partial k}{\partial x} + hv\frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left(\frac{h\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + h(P - \varepsilon) \tag{3}$$

$$u\frac{\partial \varepsilon}{\partial x} + hv\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left(\frac{h\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + h(C_1 P - C_2\varepsilon) \tag{4}$$

where

$$\nu_t = C_\mu\frac{k^2}{\varepsilon}, \qquad P = \nu_t\left(\frac{\partial u}{\partial y} - \frac{u}{hR}\right)^2$$

and σ_k, σ_ε, C_μ, C_1, and C_2 are model constants assigned the values 1.0, 1.3, 0.09, 1.43, and 1.92, respectively. Other symbols are defined in Fig. 1 and the Nomenclature.

A curvature correction is also applied to the k–ϵ model. Dash et al.[14] follow Hah and Lakshminarayana[21] by modifying the constant C_1 in the ϵ equation as follows:

$$C_1 = 1.43(1 + C_c Ri) \tag{5}$$

where C_c is an additional empirical constant for which Dash et al.[14] suggest a value of 0.16 and Ri is a Richardson number for which the Launder et al.[22] definition,

$$Ri = \left(\frac{u}{R}\right)\left(\frac{k}{\epsilon}\right)^2\left(\frac{\partial u}{\partial y}\right) \tag{6}$$

is used. Coupling between the mixing-length and k–ϵ models is set to occur at $y^+ = 50$. At this point the eddy viscosities given by the two formulations are required to be equal; this, together with the requirement that production of turbulence kinetic energy equals dissipation, yields boundary conditions for k and ϵ at the coupling point. Dash et al.[14] state that results are insensitive to the y^+ value at which coupling is set to occur, provided that this is in the log-law region. The Dash et al. model was not modified in any way for the present work.

The unmodified version of the k–ω method used in the present work was that presented by Wilcox in Ref. 18, with a curvature correction term added to the k equation. For the coordinate system of Fig. 1, the k and ω equations are

$$u\frac{\partial k}{\partial x} + hv\frac{\partial k}{\partial y} + C_k\nu_t\frac{u}{R}\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(h\left(\nu + \sigma^*\nu_t\right)\frac{\partial k}{\partial y}\right) + h(P - \beta^*\omega k) \tag{7}$$

$$u\frac{\partial \omega}{\partial x} + hv\frac{\partial \omega}{\partial y} = \frac{\partial}{\partial y}\left(h(\nu + \sigma\nu_t)\frac{\partial \omega}{\partial y}\right) + \frac{\alpha h P}{\nu_t} - \beta^* h\omega^2 \tag{8}$$

where

$$\nu_t = \frac{k}{\omega}, \qquad P = \nu_t\left(\frac{\partial u}{\partial y} - \frac{u}{hR}\right)^2$$

and α, β, β^*, σ, σ^*, and C_k are model constants assigned the values $\frac{5}{9}$, $\frac{3}{40}$, $\frac{9}{100}$, $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{9}{2}$, respectively.

The term $C_k \nu_t(u/R)(\partial u/\partial y)$ is added to the convection terms on the left-hand side of the k equation to account for curvature effects, as suggested by Wilcox and Chambers.[23] Use of this curvature correction term implies the assumption that k is the turbulent mixing energy, that is, not the exact turbulence kinetic energy but some measure of it.[18,23] In future work other approaches to correcting the k–ω model for curvature effects might also be tried, for example, modification of the dissipation-rate equation along the lines suggested by Hah and Lakshminarayana.[21] Integration from the wall is possible with the k–ω model without any need for low-Reynolds-number modifications or other special near-wall models. The k–ω method was subsequently modified, as described later, to improve predictions for wall jets.

The MS model is a simplified Reynolds stress transport model proposed by Wilcox.[19,24,25] As mentioned earlier, it is attractive in principle for wall jets because it does not rely on the eddy-viscosity hypothesis, which fails badly in the velocity-peak region of wall jets, yet it requires only about 50% more computing time than the k–ϵ or k–ω models. In the MS model the turbulent eddies are categorized into two partitions, each having a different kinetic energy level. The eddies of the upper partition are assumed to be relatively large, to contain most of the turbulence energy, to be essentially inviscid, and to be responsible for all of the Reynolds shear stresses and production of turbulence energy by interaction with the mean flow. The eddies of the lower partition are assumed to be small and isotropic and to be responsible for all dissipation of turbulence energy. These assumptions are inspired by the large-eddy equilibrium hypothesis first proposed by Townsend.[26] The MS model makes use of Eqs. (7) and (8) for k and for ω. These are supplemented by transport equations for $(k-e)$, the energy of the upper-partition eddies, and for the Reynolds stresses associated with the upper-partition eddies; e is the energy of the lower-partition eddies. A relation to model the rate of energy transfer from the upper- to the lower-partition eddies is also required. No curvature corrections were introduced into the MS model because at least some Reynolds stress transport models have been shown to have an inherent capability of capturing curvature effects.[27] The MS model has 10 closure coefficients or constants, 5 of which are shared with the k–ω model. It is fully described in Ref. 19.

Methodology

Performance of the various turbulence models was assessed simply by comparing computed predictions for streamwise variation of the wall-jet thickness and velocity parameters y_m, $y_{1/2}$, and U_m (see Fig. 1) as well as skin-friction coefficient C_f and profiles of mean velocity u, turbulence kinetic energy k, and Reynolds shear stress $\overline{u'v'}$ against available experimental data. Computations were carried out using a code developed specifically for this work.

In most applications, and in all available experimental data, the wall-jet region of the flowfield is relatively thin, and streamwise diffusion is negligible relative to cross-stream diffusion. For present purposes a parabolized thin-shear-layer computational algorithm was, thus, satisfactory, and the fast solution capability of such algorithms was important in view of the very large number of flow-development computations that the work entailed. A steady incompressible two-dimensional parabolized thin-layer Navier–Stokes code was developed. A curvilinear surface-normal coordinate system (see Fig. 1) was adopted so that highly curved flows could be accommodated. In this coordinate system the equations of motion are

$$\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}(hv) = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + hv\frac{\partial u}{\partial y} + \frac{vu}{R} = -\frac{\partial}{\partial x}\left(\frac{p}{\rho} + \overline{u'^2}\right) - h\frac{\partial(\overline{u'v'})}{\partial y} - \frac{2}{R}\overline{u'v'} + hv\frac{\partial}{\partial y}\left\{\left(\frac{\partial(hu)}{\partial y} - \frac{\partial v}{\partial x}\right)\Big/h\right\} \tag{10}$$

$$\frac{u^2}{hR} = \frac{\partial}{\partial y}\left(\frac{p}{\rho} + \overline{v'^2}\right) \tag{11}$$

We assumed $\overline{u'^2} = \overline{v'^2} = 2k/3$ except in the MS model, where these stresses are explicitly given by the model equations.

All equations were nondimensionalized for use in the computer code. The finite volume discretization scheme described by Patankar[28] was used with an implicit unconditionally stable streamwise marching algorithm. Grid spacing in the wall-normal y direction increased geometrically, with a multiplier in the range 1.05–1.1; the first grid point out from the wall was always at $y^+ < 1$, and there were typically 150–200 grid points in the y direction. The streamwise static pressure distribution along the outer edge of the wall jet flows was specified from the experimental data; for curved flows the radial pressure distributions were determined from the radial momentum equation (11). The code was verified against exact laminar boundary-layer and wall-jet solutions, against published solutions using the Baldwin–Lomax[16] and Dash et al.[14] turbulence models, and against solutions given by Wilcox's EDDYBL code[13] for the k–ω and MS models. Grid refinement studies were done to ensure grid independence of the results.

For the k–ϵ, k–ω, and MS methods a Dirichlet-type boundary condition was applied at the outer edge of the shear layer, as recommended by Wilcox (Ref. 13, Chap. 7). The necessary equations are obtained from the k, $(k-e)$, ϵ, and ω transport equations by setting $\partial/\partial y$ terms and production terms to zero; this yields ordinary differential equations that were integrated from the starting values by a Runge–Kutta routine to give the edge values of k, $(k-e)$, ϵ, and ω at each streamwise station. The wall boundary condition for ω took the form suggested by Wilcox (Ref. 13, Chap. 4), with the nondimensional roughness height k_R^+ set to 0.01 because only smooth-wall cases were calculated.

The k–ω method is known to be sensitive to the value of ω in the freestream.[29] Different starting values for ω_{edge} were tried for some of the self-similar flows that were computed, with no significant changes to the final self-similar results produced by either the k–ω or the MS method. The computed k profiles of nonsimilar flows exhibited moderate sensitivity to the starting value of ω_{edge}. Starting values of order 10 were generally used for $\omega_{\text{edge}} L_{\text{ref}}/u_{\text{ref}}$, with some adjustment to obtain reasonable initial k vs y profiles. The sensitivity to ω_{edge} was only a minor difficulty in the present work, consistent with the finding that the sensitivity is far more pronounced for free shear flows than for wall-bounded flows.[29,30]

Time constraints permitted development of modifications for only two of the turbulence models. For reasons mentioned in the Introduction, the k–ω and MS models were chosen for modification. The experimental data used for modification or tuning purposes included plane wall jets in still air, plane wall jets in adverse pressure gradient, and curved (log-spiral) wall jets in still air. All of the tuning flows are equilibrium or self-similar flows. This is convenient because nondimensional flow parameters and profiles have the same values and shapes at all streamwise positions along such flows and difficulties with starting flow-development computations are minimal. The tuning flows were the following: Wilson's[31] plane wall jet in still air (Wil); Irwin's[32] and Gartshore and Newman's[33] plane wall jets in moving streams with $U_e/U_m = 0.38$ and 0.77, respectively (Ir and GaN, respectively); and Guitton and Newman's[34] curved still-air wall jets over log spirals with $x/R = 0.67$ and 1.0, respectively (GuN1 and GuN2).

Modification or tuning of the k–ω and MS models followed the approach of Wilcox (Ref. 13, Chap. 4). The form of the k and ω equations was not altered; only the model constants were adjusted in the tuning process. The form of the additional equations that appear in the MS model was also not altered in the initial modified version, MS1. Three matching points were chosen for purposes of tuning: the maximum shear stress point, the zero shear stress point and the law-of-the-wall region. When the turbulence model equations are applied at these matching points, a number of terms drop out. The simplified equations thus obtained give relations between the various constants of the turbulence model and measurable flow parameters. The set of equations is not fully determinate, and so some of the constants are free and their values were found by trial-and-error optimization. The values were chosen to match the experimental growth rate, $dy_{1/2}/dx$, and, insofar as possible, the skin-friction coefficient C_f and the profiles of u, k and $\overline{u'v'}$. Because all curvature corrections vanish for plane flows, these were dealt with first, and then the additional constants for curvature effects were determined by comparisons with data for curved wall jets.

The available data for self-similar wall jets show that k, $\overline{u'^2}$, $\overline{v'^2}$, and $\overline{u'v'}$ all attain a fairly broad maximum at approximately the same nondimensional distance from the wall, $y/y_{m/2} \approx 0.7$. Thus, at this point, the maximum shear stress point, the turbulence model equations simplify considerably due to dropping out of $\partial/\partial y$ terms. At the zero shear stress point, the equations also simplify because the production terms drop out. Unfortunately, this point is quite close to the wall, and the available data here are neither abundant nor accurate. In the log-law region one can assume that production equals dissipation, again giving simpler equations. There is still some controversy over whether or not the universal logarithmic velocity distribution exists near the wall in wall jets but there is now good evidence that it does in at least some conditions (e.g., Wygnanski[35]), albeit only for y^+ less than about 100. In the present work it was assumed that the log law does apply, with the same values of the constants as in conventional boundary layers. The reduced equations at the three matching points for the k–ω and MS models are available in Ref. 36. In the case of the k–ω model, the tuning analysis yielded four relations between the model constants and flow parameters that could be directly determined from the experimental data for the tuning flows; this left two free constants, σ^* and C_k, which had to be determined by a trial-and-error process. For the MS model there were seven relations, leaving three free constants.

Results and Discussion

Calculations with the original versions of all four turbulence models were done for conventional boundary layers [flat plate, adverse pressure gradient,[37] and convex wall (flow 0233, Ref. 4)], as well as for the aforementioned wall jet flows.

For the three conventional boundary layers, including that on the convex surface, all four turbulence models produced quite similar predictions of boundary-layer thickness, shape factor, C_f, and

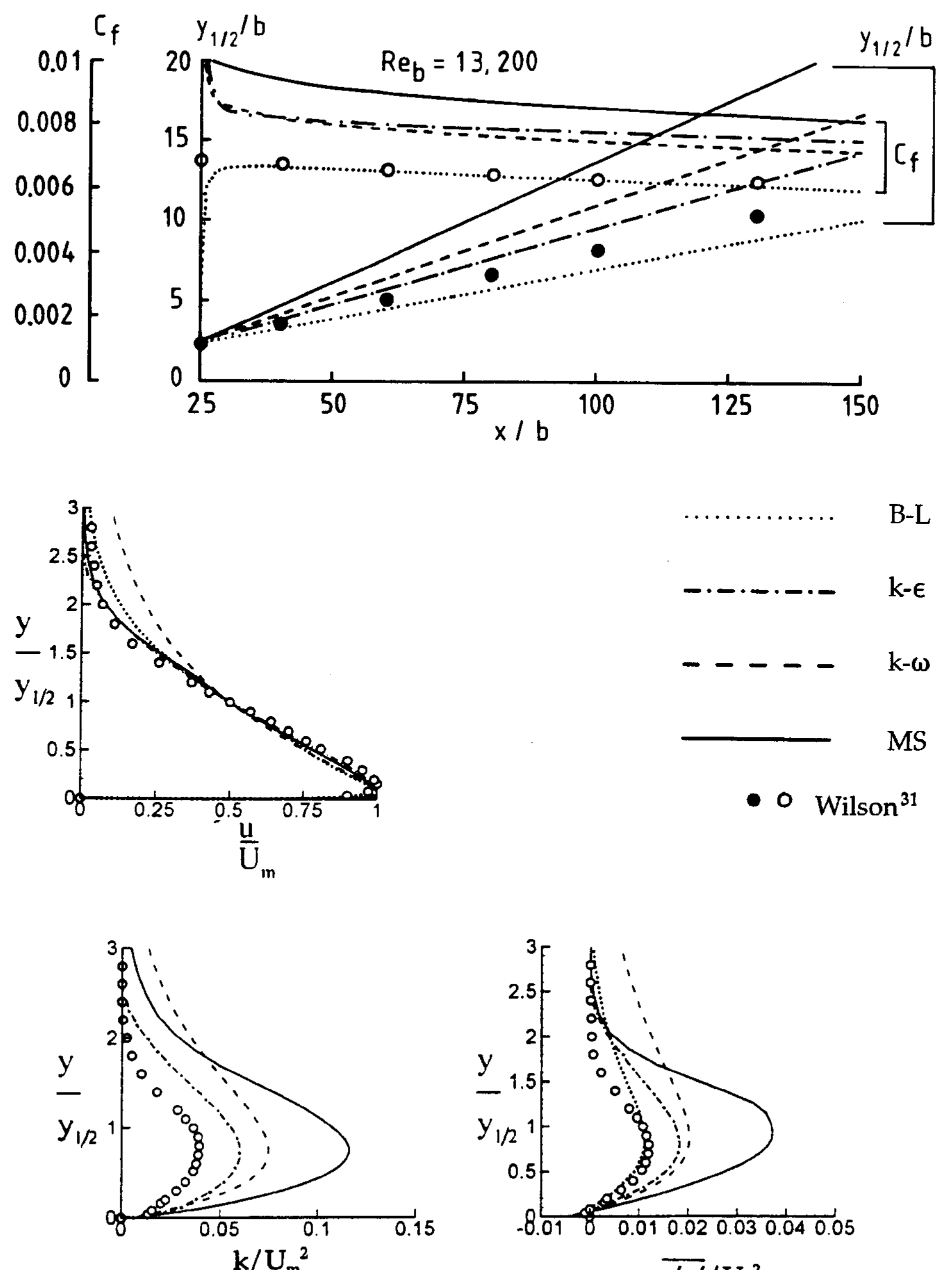

Fig. 2 Comparison between experiment and predictions using unmodified turbulence models: plane wall jet in still air (flow Wil).

velocity profiles, and the predictions generally agreed reasonably well with experiment. This is to be expected because the turbulence models and the curvature corrections that they incorporate were developed for this type of flow. For the adverse-pressure gradient and convex-surface boundary layers, however, the k profiles were not well predicted by any of the models (~30% error); the k–ω and MS models did, on the other hand, give good predictions of the $\overline{u'v'}$ profiles. Overall, the performance of the k–ω and MS models was about equal and was noticeably superior to the other models for the conventional boundary-layer flows. It is interesting that the MS model performed very well for the convex boundary layer, supporting the hope that it could capture curvature effects without introduction of curvature corrections.

The performance of the four turbulence models varied widely for the still-air wall jets. Predictions and experiment are compared in Figs. 2 and 3 for the plane and log-spiral wall jets, flows Wil and GuN1, respectively. For reasons touched on in the next paragraph, flow development predictions can be adequately assessed by considering only plots of the thickness parameter $y_{1/2}$ and of C_f vs x. In Figs. 2 and 3, it is seen that the k–ω and MS methods give the poorest predictions of development of the still-air wall jets and also the poorest predictions of profiles of velocity and turbulence parameters. The excessive velocities predicted by the k–ω model for $y > 1.5y_{1/2}$ were also observed by Gerodimos and So[38] in their calculations of plane wall jets with the k–ω model. For the curved flow of Fig. 3, the MS method produces particularly poor profiles, with unrealistic kinks. Note that the kinks occur in the mixing layer, well outside the wall-dominated zone of the flow. Results for flow GuN2 are similar to those in Fig. 3.

Wall jets are quite different from conventional boundary layers in that the flow is dominated by the intense mixing between the jet and the ambient or freestream air. The increase of the thickness parameter $y_{1/2}$ is mainly determined by the mixing, and this in turn mainly determines the rate of decay of the peak velocity U_m, by virtue of conservation of momentum. Skin friction has relatively little impact on thickness growth and peak-velocity decay and is only important if flow separation is a possibility. Incidentally, this implies that if predictions of $y_{1/2}$ are good, those for U_m will also

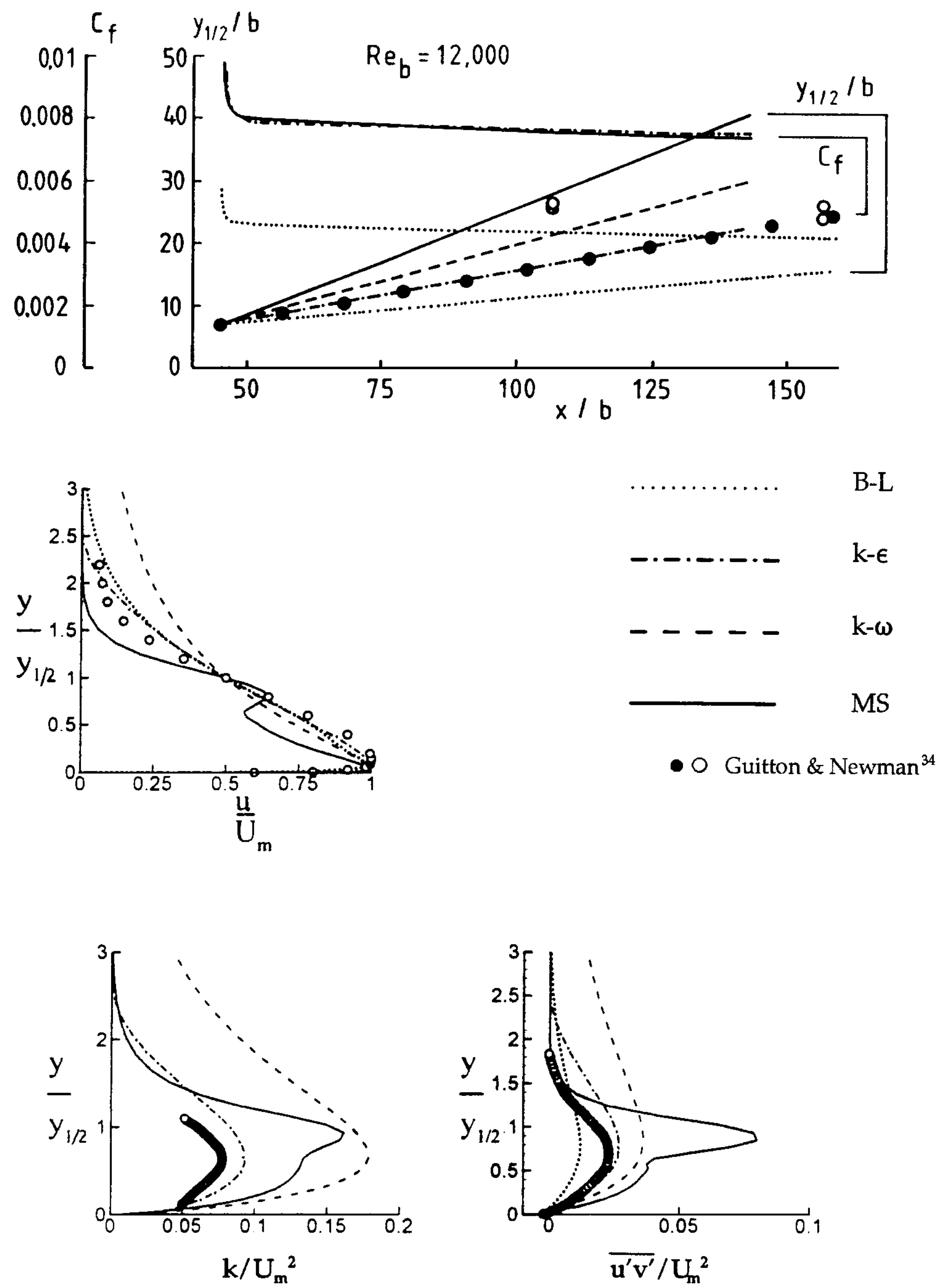

Fig. 3 Comparison between experiment and predictions using unmodified turbulence models: wall jet over a log spiral in still air (flow GuN1).

be good. Furthermore, large energetic eddies are associated with the mixing in the outer layer ($y > y_m$), and these impinge on the flow in the inner layer ($y < y_m$) and strongly affect the turbulence structure there. It is not surprising, therefore, that the k–ω and MS models produce poor predictions for wall jets because they have not been tuned for this type of flow. However, with the exception of the curvature corrections that it incorporates, the Dash et al.[14] k–ϵ model has also not been tuned for wall jets, and it is encouraging to see that it gives at least fair predictions for the wall-jet flows. Its superior performance for these cases reflects the relatively good performance of k–ϵ models for free shear layers as well as the proven performance of the mixing-length model used by Dash et al. for the near-wall zone of conventional boundary layers. Recall that the Baldwin–Lomax[16] model used herein only incorporates a curvature correction in the inner flow zone; it is clear from Fig. 3 that it would benefit from a curvature correction in the outer zone as well. Note that in the outer layer of wall jets on convex surfaces the radial gradient of angular momentum is negative, corresponding to instability to radial perturbations; turbulent mixing is known to be much enhanced as a result, and large curvature corrections are necessary for simple turbulence models.

The Baldwin–Lomax[16] and k–ω models give very good predictions for the plane wall jets in adverse pressure gradient, flows Ir and GaN; the predictions produced by the other two models are considerably poorer but still fairly good. Figure 4 shows results for flow Ir and results for flow GaN are comparable. It is clear that the turbulence models have less difficulty with these flows than with the still-air wall jets. This can be explained in terms of the ratio U_m / U_e, which has values of infinity for the still-air wall jets, vs 2.63 and 1.3 for flows Ir and GaN. Thus, the mixing in the outer layer is less dominant in the latter flows, which are, thus, less different from the conventional boundary layers for which the turbulence models were originally developed.

It was hoped that with suitable adjustment of model constants the performance of the k–ω and MS models for wall-jet flows could be made equivalent to their superior performance for conventional

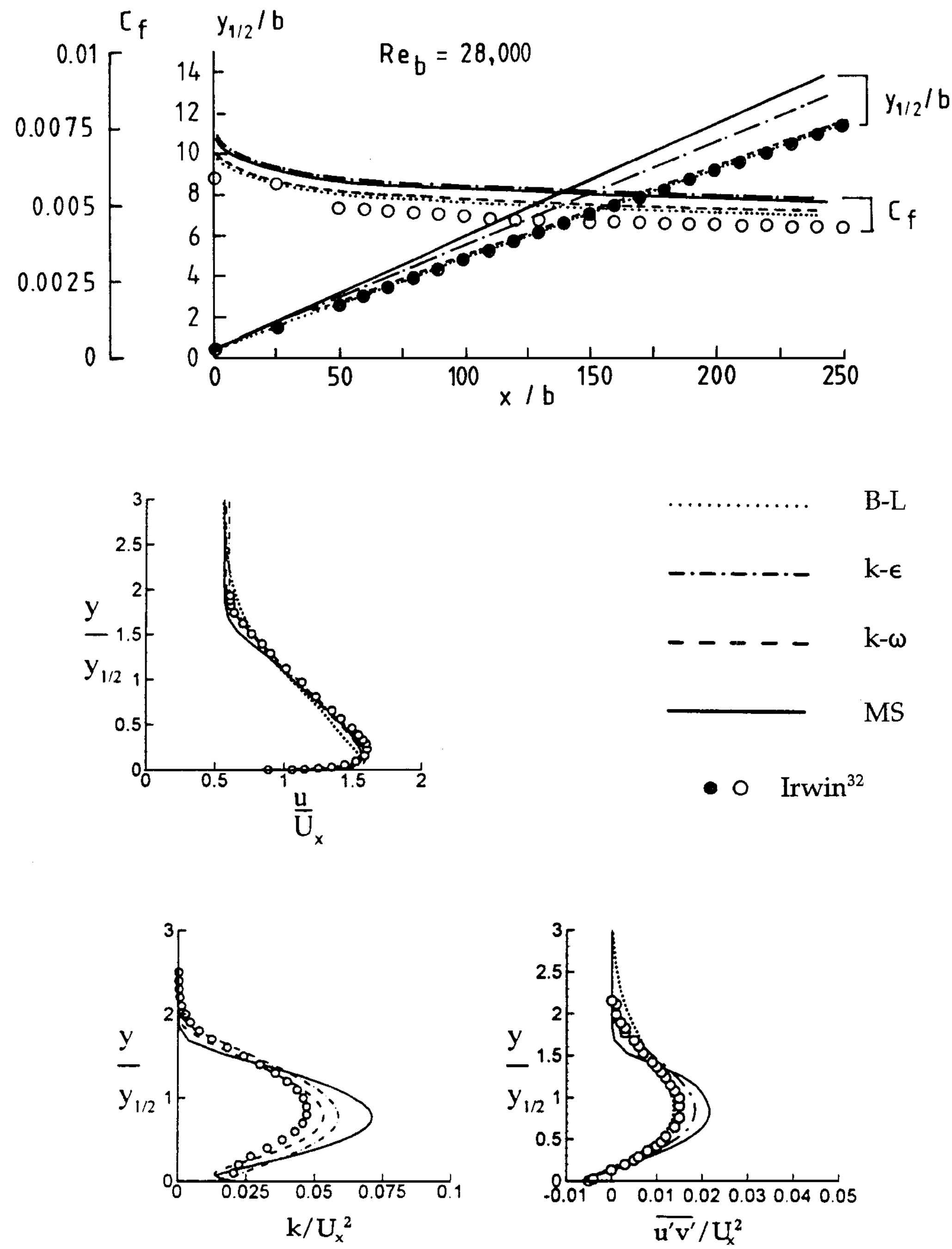

Fig. 4 Comparison between experiment and predictions using unmodified turbulence models: plane wall jets in adverse pressure gradient (flow Ir).

boundary layers. The methodology used for tuning the models was outlined earlier. The models were tuned for each individual tuning flow; that is, an optimum set of model constants was determined for each of the tuning flows.

Figure 5 shows predictions of the modified k–ω method for the still-air log-spiral wall jet, flow GuN1. The modifications produce a substantial improvement over the predictions of the original method seen in Fig. 3. The growth $y_{1/2}$ vs x is very well predicted; this is ensured by the tuning methodology, which allows quite direct control of the growth rate prediction. The predicted velocity profile shape is good, and the profiles of turbulence parameters are fairly good. Unfortunately, C_f is still poorly predicted, resulting in some inaccuracy in decay of U_m. Predictions for the more strongly curved log-spiral wall jet (flow GuN2) are somewhat poorer than those seen in Fig. 5, whereas those for the other tuning flows are better. The quality of the predictions produced by the modified k–ω model relates directly to the intensity of mixing in the outer layer of the wall jet flows. This mixing is more intense for higher values of U_m / U_e and for stronger convex curvature. As the outer-layer mixing becomes more intense it exerts more influence on the inner-layer turbulence, the overall turbulence structure in the shear layer differs more from that in conventional boundary layers, and, even with tuning, the k–ω model is less able to fully represent flow behavior.

Although modification of the k–ω model was successful in the sense that it could be tuned to give good predictions of each of the tuning flows, the values of the tuned model constants, listed in Table 1, differed substantially from one tuning flow to another, despite efforts to maintain commonality of values. Unfortunately the predictions, especially for the still-air wall jets, are very sensitive

Table 1 Model constants for the original and modified k–ω models for each tuning flow

Constants	Original k–ω	Modified k–ω Wil	Ir	GaN	GuN1	GuN2
C_k	4.5	——	——	——	1.05	1.417
σ^*	0.5	0.36	0.7	1	0.36	0.36
σ	0.5	0.268	0.477	0.647	0.289	0.312
β^*	0.09	0.114	0.11	0.128	0.133	0.155
β	0.075	0.177	0.17	0.199	0.207	0.24
α	0.556	1.417	1.308	1.246	1.417	1.417

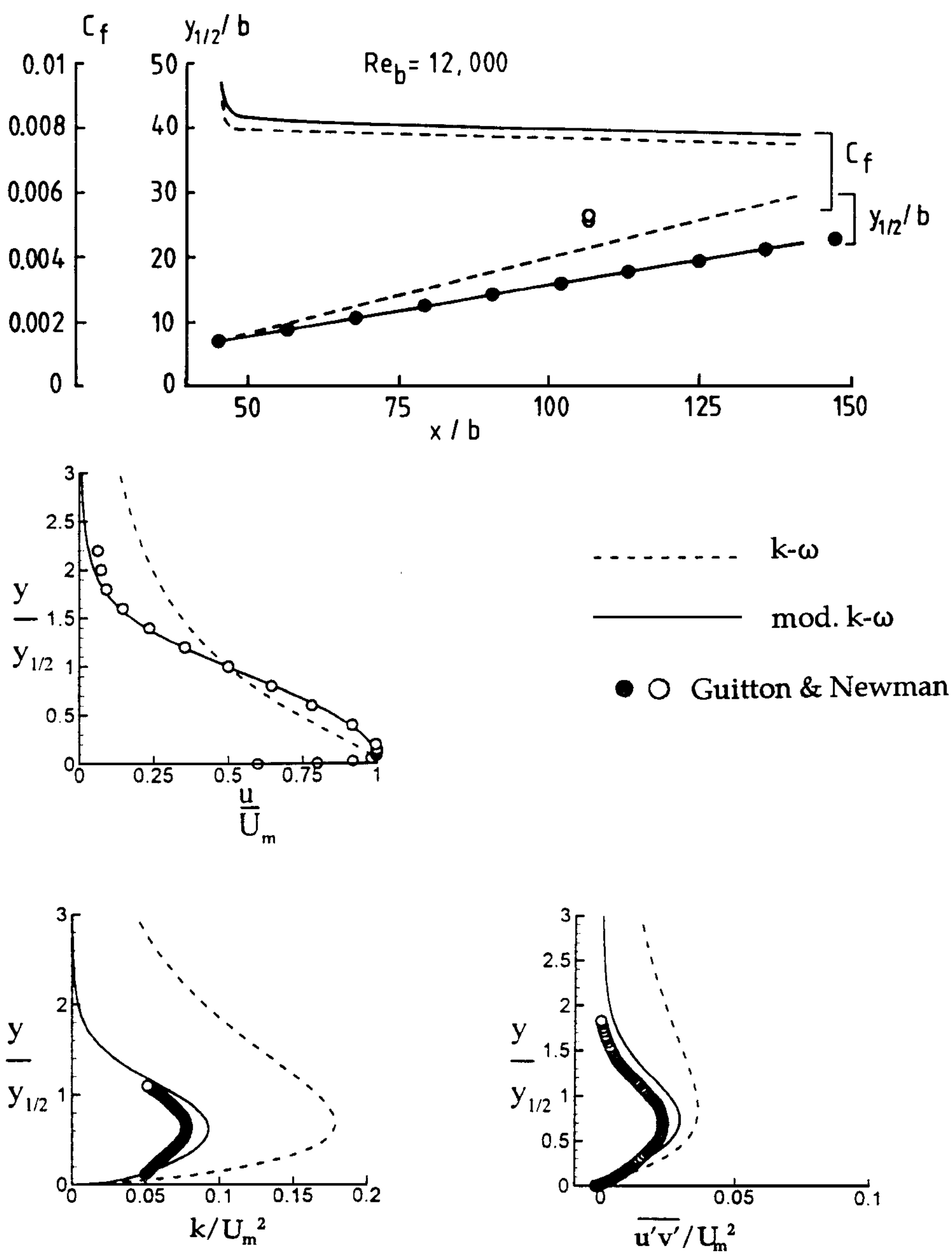

Fig. 5 Comparison between experiment and predictions using the original and modified k–ω turbulence models: wall jet over a log spiral in still air (flow GuN1).

to values of the model constants, and no single set of values giving reasonably good predictions for all of the wall-jet flows was found. Another approach would be to attempt to find correlation relations between the model constants and one or more simple flow parameters such as U_m/U_e. Of course, for conventional boundary layers, the correlations would revert to the established values of the constants. In the present work correlations were established between the constants and the rather complex flow parameters that arise in the tuning process. Preliminary attempts to compute nonequilibrium wall-jet flows using these correlations were unsuccessful. The difficulty experienced in finding satisfactory sets of values of the k–ω model constants for still-air wall jets, and the sensitivity to these values, may reflect the Wilcox (1988) k–ω model's inferiority, relative to k–ϵ models, for free shear flows.[18] Wilcox (Ref. 13, 2nd ed., pp. 119–122) has recently presented an improved version of the k–ω model that offers much improved performance for free shear flows; it would be interesting to assess this model for wall jet flows.

Figure 6 shows predictions of the MS1 modification (no changes to model form) of the MS method for the still-air log-spiral wall jet, flow GuN1. Excellent prediction of the growth $y_{1/2}$ vs x is enforced by the tuning process. Except for C_f, other predictions are also substantially better than those of the original MS method, seen in Fig. 3. In particular, the profiles no longer have kinks. Nevertheless, the predicted velocity profile in Fig. 6 is still rather poor. The same problems were seen, to a somewhat greater extent, for the more strongly curved wall jet, flow GuN2. The predictions for the other three tuning flows were somewhat better than those seen in Fig. 6, but all left something to be desired. In fact, the velocity profile shape predicted by the MS1 model for the plane wall jet in still air was actually inferior to that predicted by the original MS method. Overall, the predictions of the MS1 model, particularly for velocity profiles, were inferior to those of the modified k–ω model. This is somewhat surprising given that the model was tuned to each of the flows individually and that there are more model constants (10 vs 6) that can be tuned. As with the k–ω model, there are substantial variations in the optimum values of the model constants from flow to flow, and no single set giving reasonably good predictions of all the tuning flows was found.

Three additional modifications to the form of the MS model were tried because modification of only the model constants did not produce satisfactory results. These modifications were prompted by the observation that calculated production and dissipation of turbulence energy were not in balance as they should be for the tuning flows, all of which were equilibrium flows. The modifications dealt with the upper-partition energy fraction $(k-e)/k$. Although they produced

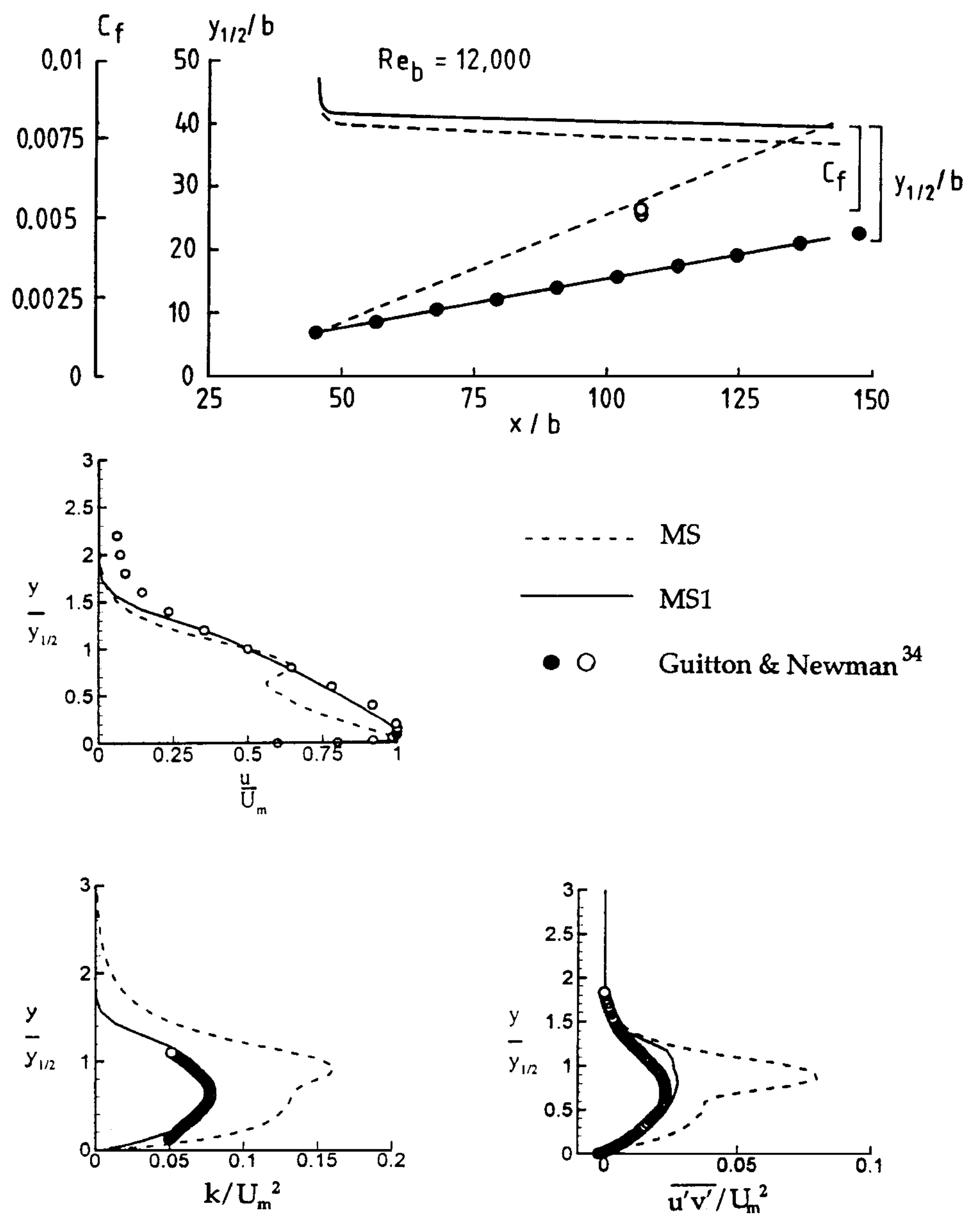

Fig. 6 Comparison between experiment and predictions using the original MS and modified MS1 turbulence models: wall jet over a log spiral in still air (flow GuN1).

significantly different velocity, etc., profiles from MS1, the three additional modified versions produced predictions of approximately the same quality as MS1.

A full description of the work is available in Ref. 36.

Conclusions

Four turbulence models, Baldwin–Lomax,[16] Dash et al. k–ϵ (Ref. 14), Wilcox k–ω (Ref. 18), and Wilcox MS,[19] have been assessed for predicting development of plane and curved turbulent wall-jet flows. Whereas all of the models exhibited significant deficiencies, the Dash et al.[14] k–ϵ model performed reasonably well in an overall sense, particularly for the curved wall jets in still air. The deficiencies of the other three models were particularly severe for these flows. The overall effectiveness of the Baldwin–Lomax[16] model could be easily improved by incorporating a curvature correction for the eddy viscosity in the outer layer.

Two of the models, k–ω and MS, were modified using a matching-point approach. After tuning to individual flows, the modified methods gave reasonably good predictions of thickness growth and decay of peak velocity. The modified k–ω model also gave fairly good predictions of velocity and turbulence-parameter profiles, but predictions of C_f were still poor in some cases, notably for the curved wall jets in still air. The MS model, even after modification, gave poor predictions of velocity profiles for all of the still-air wall jets and also poor predictions of C_f for the curved still-air wall jets. Most disappointing is the finding that optimum values of the k–ω and MS model constants varied substantially from one wall-jet flow to another.

Wall jets in an external stream of relatively high velocity have less intense mixing in their outer layer and are relatively easy to predict. All four turbulence models, in both their original and modified forms, gave reasonable predictions for these cases, with Baldwin–Lomax[16] and k–ω performing particularly well.

Either the Baldwin–Lomax[16] or a version of the k–ϵ model similar to that of Dash et al.[14] appear best suited at this time for engineering calculations involving wall-jet flows. If flow curvature is present, a curvature correction should be introduced into the outer zone of the Baldwin–Lomax model. Predictions of the Dash et al.[14] model could probably be improved if some effort were devoted to better tuning it for plane wall-jet flows.

Acknowledgments

This work was supported by a research grant awarded by the Natural Sciences and Engineering Research Council of Canada. The first author also gratefully acknowledges a scholarship awarded by the Canadian International Development Agency during the course of the work.

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